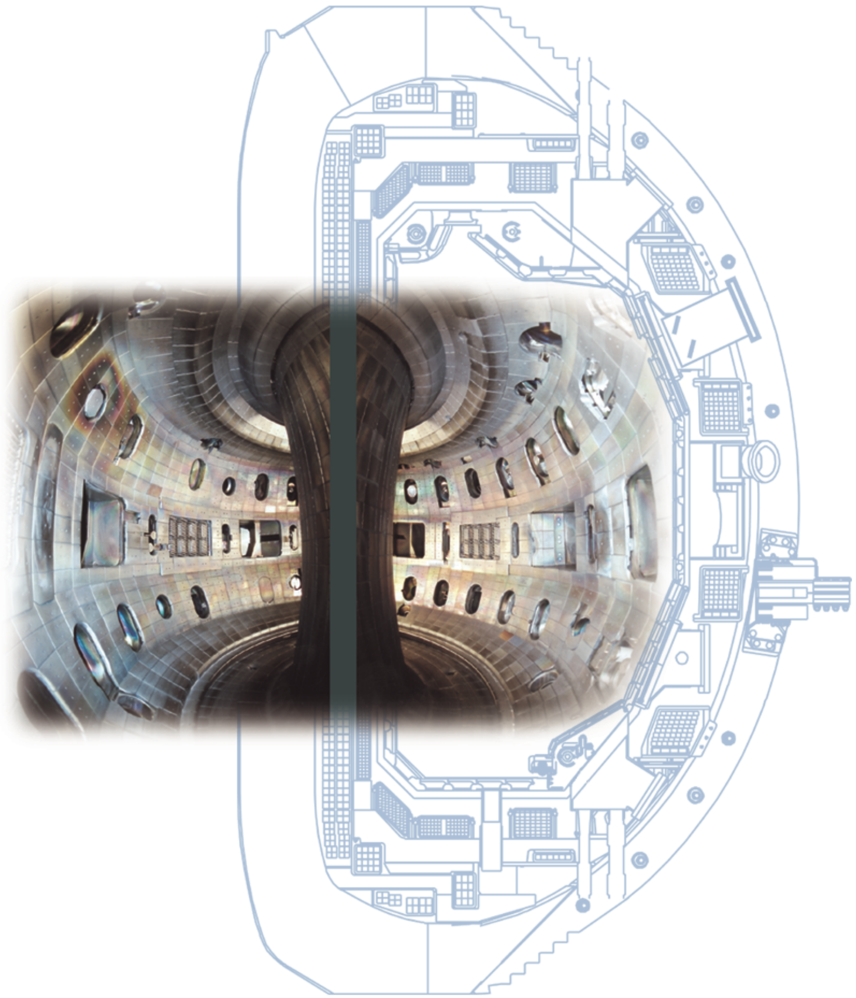
Modeling of the linearized control response of plasma shape and position has become fairly routine in the last several years. However, such response models rely on the input of accurate values of model parameters such as conductor and diagnostic sensor geometry and conductor resistivity or resistance. Confidence in use of such a model therefore requires that some effort be spent in validating that the model has been correctly constructed. We describe the process of constructing and validating a response model for NSTX plasma shape and position control, and subsequent use of that model for the development of shape and position controllers. The model development, validation, and control design processes are all integrated within a Matlab-based toolset known as TokSys. The control design method described emphasizes use of so-called decoupling control, in which combinations of coil current modifications are designed to modify only one control parameter at a time, without perturbing any other control parameter values.

System Modeling, Validation, and Design of Shape Controllers for NSTX

M.L. Walker, D.A, Humphreys, N.W. Eidietis, J.A. Leuer, A.S. Welander
General Atomics

E. Kolemen
PPPL



2011 APS/DPP
Salt Lake City, Utah
November 14-18, 2011

DIII-D NATIONAL FUSION FACILITY

GENERAL ATOMICS

Overview

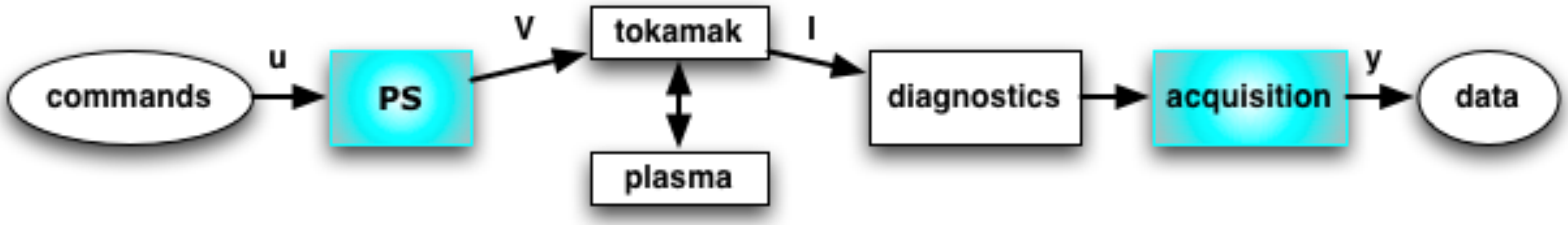
- Description of model validation purpose and process
- Results of validation tests for NSTX systems involved in plasma boundary control
- Analysis of controllability of plasma boundary in NSTX

Model-based controllers promise to improve control and therefore physics performance

- **Mathematical models can be exploited by modern model-based control design techniques, BUT ...**
  - Generated controllers are only useful if models used are **predictive** of actual system behavior.
- **Testing controllers using simulation with predictive models provides confidence in operational deployment.**
- **Predictive models can only be assured through validation of models with data.**

Validation Process

- **Break down model into component parts**



- **Identify or collect data for validation**
  - Combine data gathering for multiple components whenever possible.
- **Many components represent linear processes**
  - Plasma is linearized around reference equilibrium
  - Power Supplies (PS) can have extensive nonlinearity

Most components can use generic models

- **Device**

$$\begin{bmatrix} M_{cc} & M_{cv} \\ M_{vc} & M_{vv} \end{bmatrix} \begin{bmatrix} \dot{I}_c \\ \dot{I}_v \end{bmatrix} + \begin{bmatrix} R_c & 0 \\ 0 & R_v \end{bmatrix} \begin{bmatrix} I_c \\ I_v \end{bmatrix} = \begin{bmatrix} V_c \\ 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & M_{cp} \\ 0 & 1 & M_{vp} \end{bmatrix} \begin{bmatrix} V_{cp} \\ V_{vp} \\ \dot{I}_p \end{bmatrix}$$

- **Plasma (X objects model δψ due to plasma motion)**

$$L_p^* \dot{I}_p + R_p I_p = V_{n.o.} - M_{pc}^* \dot{I}_c - M_{pv}^* \dot{I}_v$$

$$\begin{bmatrix} V_{cp} \\ V_{vp} \\ \dot{I}_p \end{bmatrix} = \begin{bmatrix} X_{cp} \\ X_{vp} \\ \mathbf{I} \end{bmatrix} \dot{I}_p + \begin{bmatrix} X_{cc} & X_{cv} \\ X_{vc} & X_{vv} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{I}_c \\ \dot{I}_v \end{bmatrix}$$

- **Diagnostics (most)**

$$\delta y = C_{I_s} \delta I_s + C_{I_p} \delta I_p$$

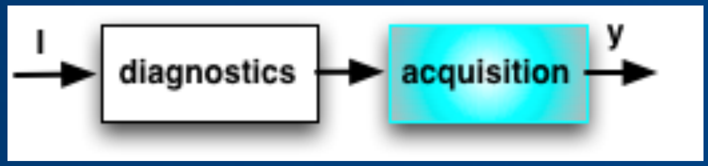
- **PS and acquisition models always machine-specific**

Vacuum response data can be used for validation of multiple (linear) components

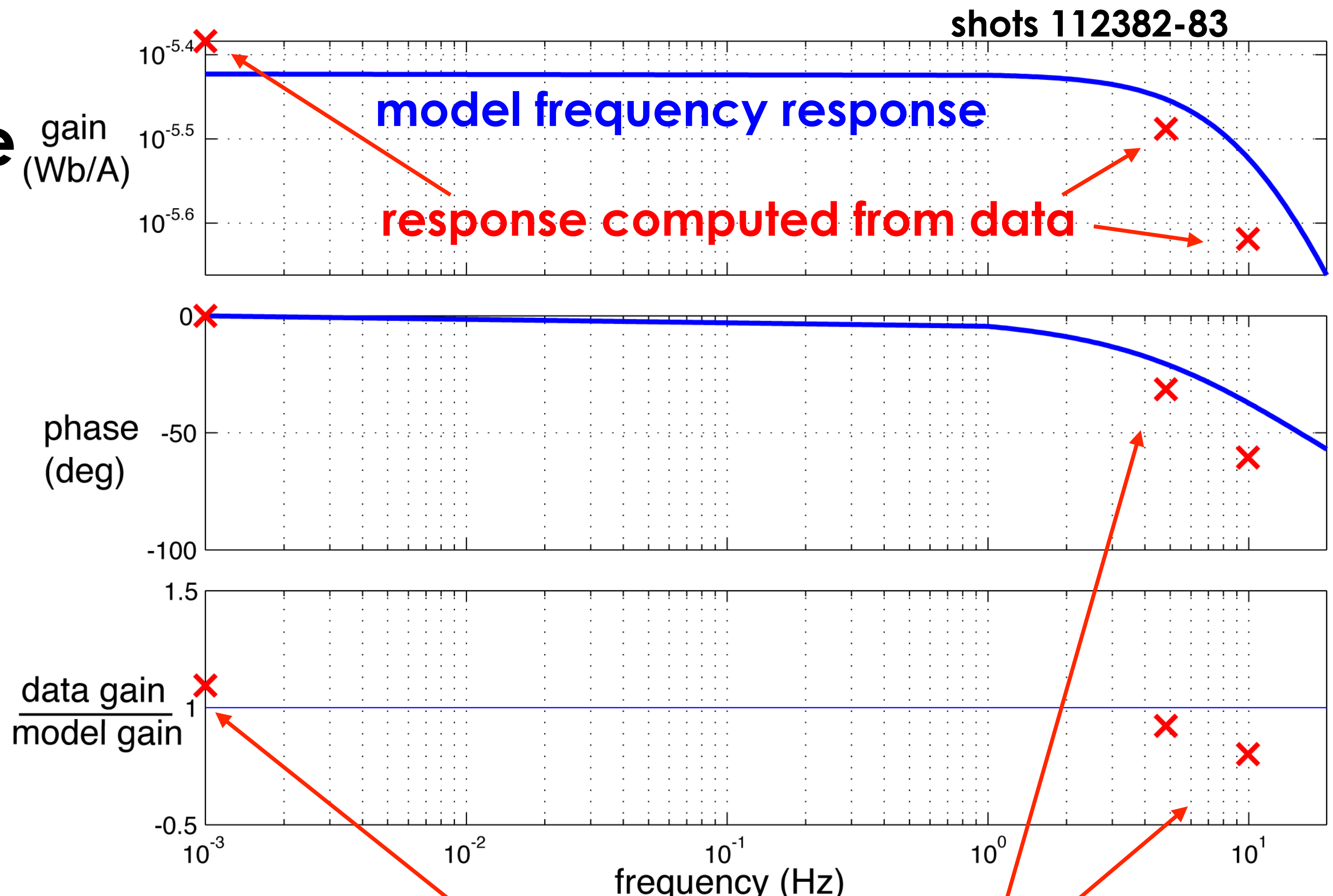
- **Data relatively easy to collect**
- **Enables complete validation of some components**
- **DC data for individual coils**
  - Constant currents provide data for diagnostic Green function validations
  - Constant voltages provide data for power supply gain
- **Multiple frequency data for individual coils**
  - If power supply nearly linear, can use sinusoidal PS command
  - Otherwise, use coil current control to produce sinusoidal coil current
- **Nonlinear power supplies can require custom data**

SYSTEMS MODEL VALIDATION

NSTX Diagnostic Validations



- **Example: flux loop response to PF1AU current**
- **Note: response includes effect of passive conductors.**

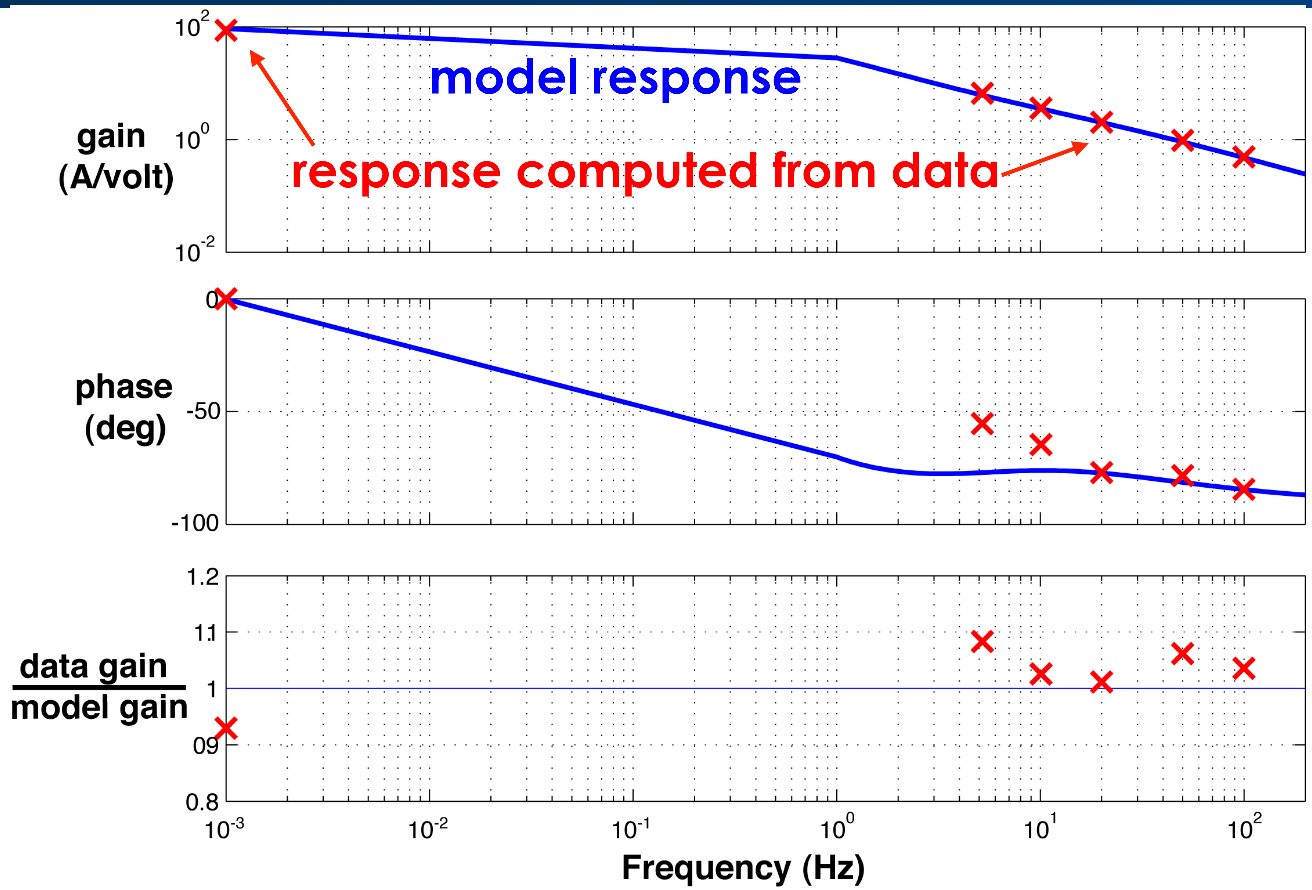


- Systematic (all diagnostics) ~10% DC gain error for certain coils
- Systematic (all coils and all diagnostics) attenuation/phase lag

=> Suggests non-ideal acquisition circuits

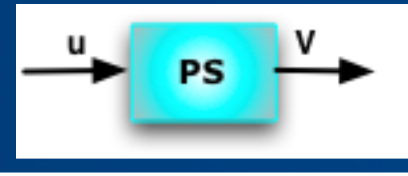
Validation of NSTX device response

Example:
PF3U
V->I
response



- Note this validation only possible for coil currents (passive conductor currents not measured).

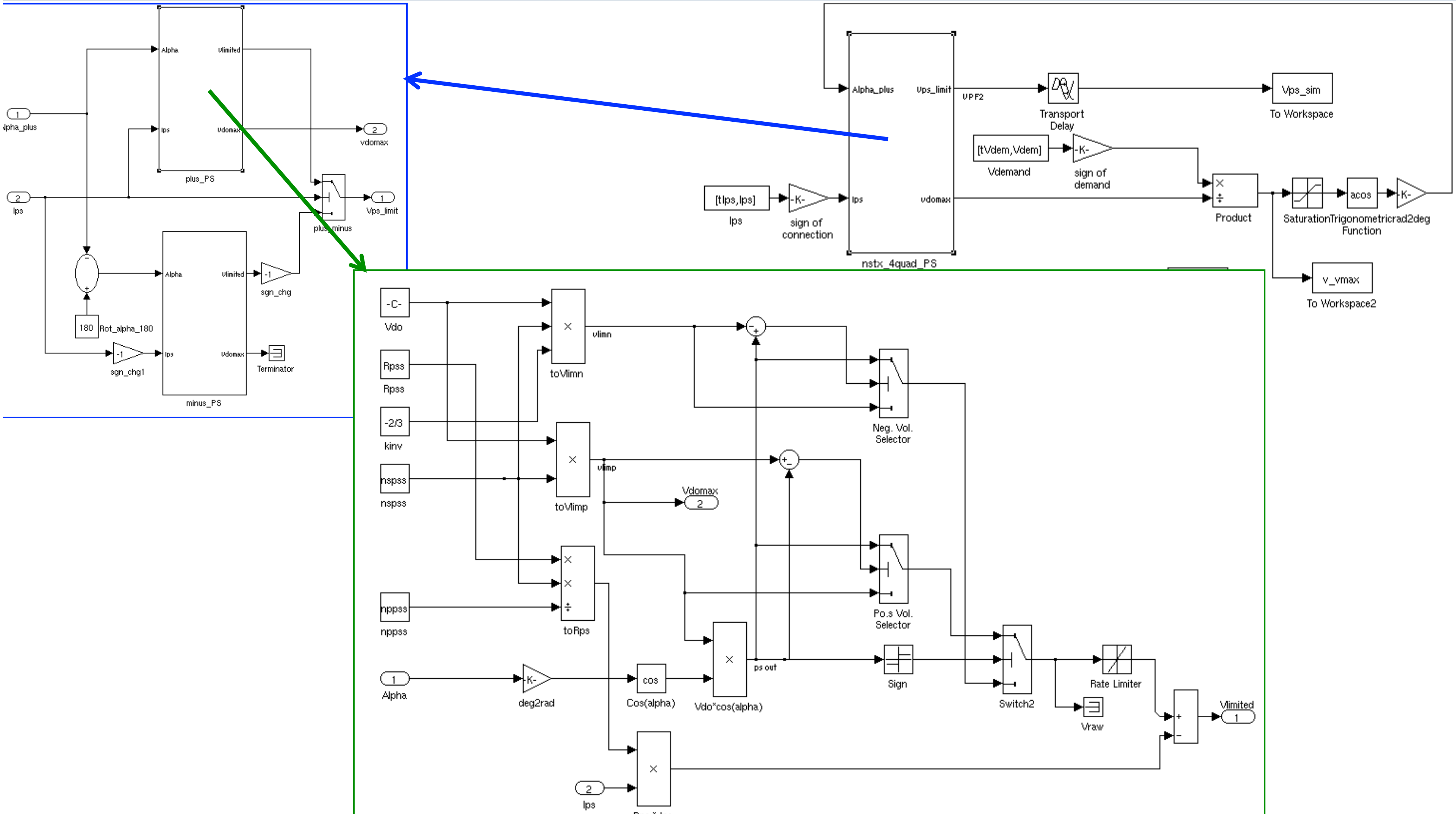
NSTX Power Supply Model



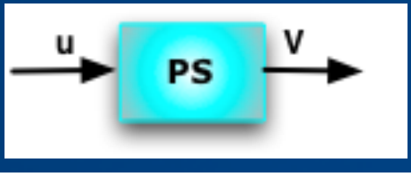
- **Parameterized power supply model provided by Ron Hatcher (PPPL).**
- **Model is reasonably predictive when correct parameters used**
  - Majority of problems due to incorrect or no information regarding model parameters.
- **Some improvements and uncertainties remain.**

For model structure, look behind ⟶

NSTX Power Supply Model



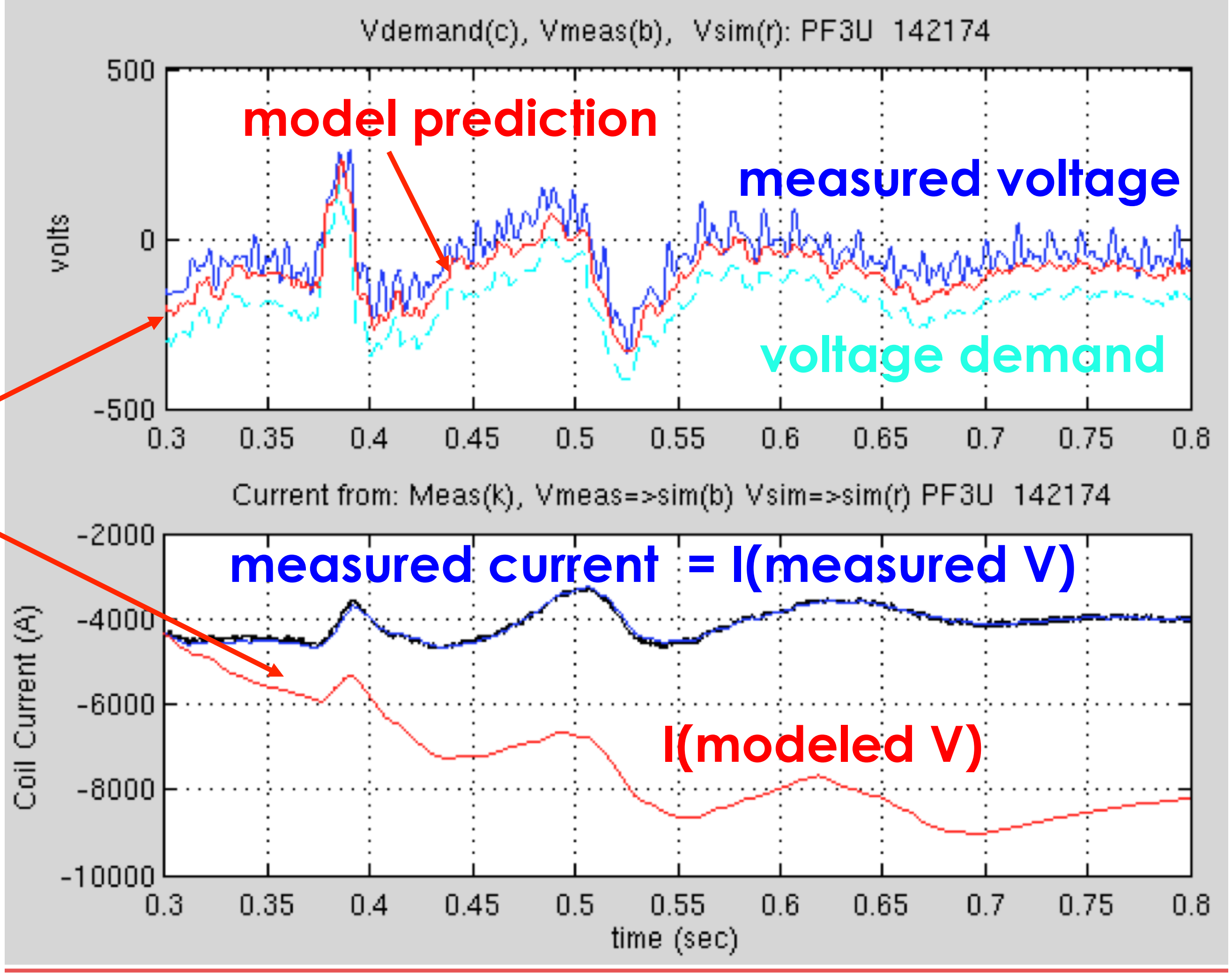
NSTX Power Supply Validations



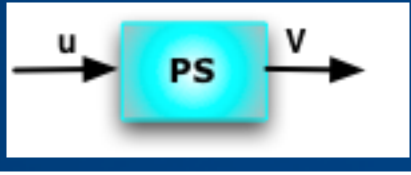
Reasonable prediction, but ...

Small predicted voltage offset can lead to drift in predicted coil current

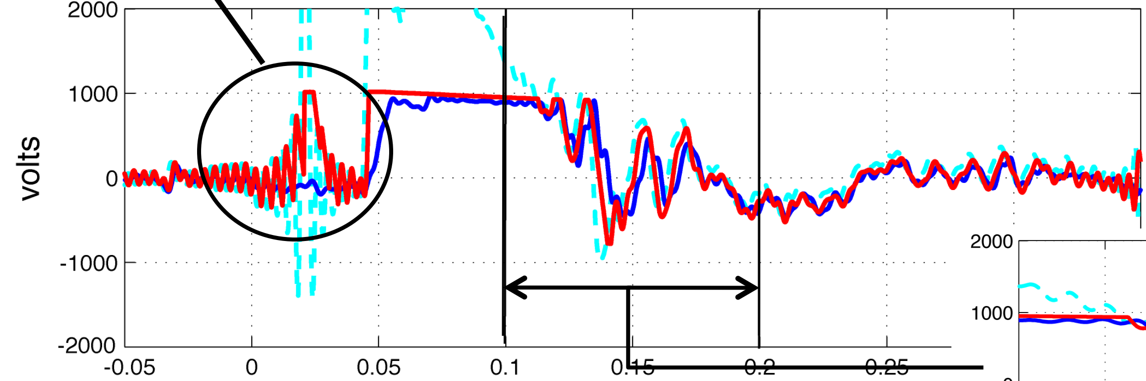
... still very useful model, as long as we are aware of limitations.



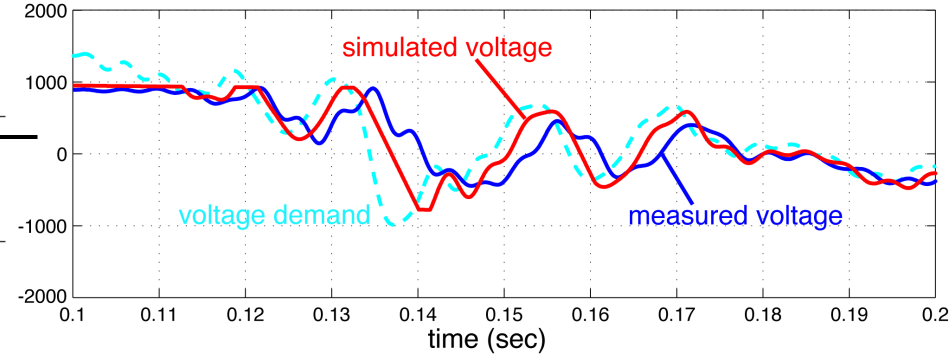
Some (PS) mysteries remain

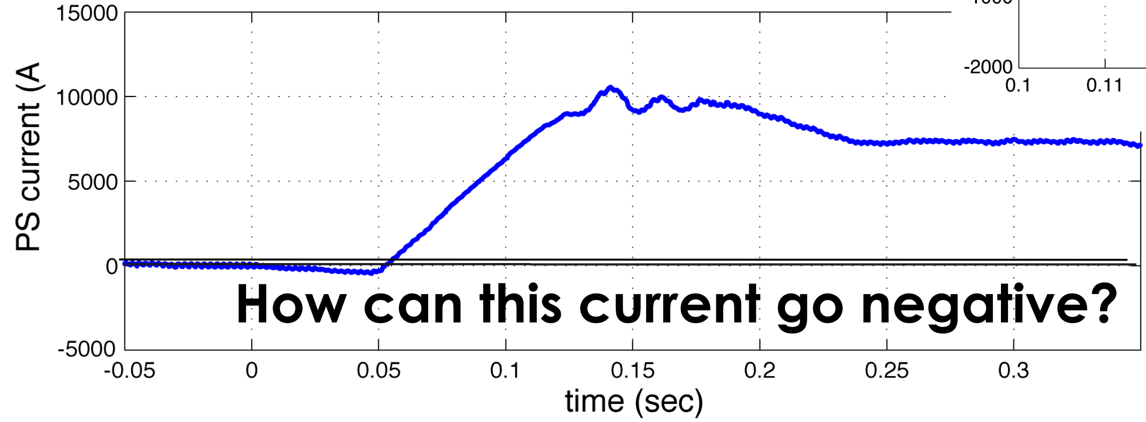


What's this?



Where is the extra phase lag coming from?



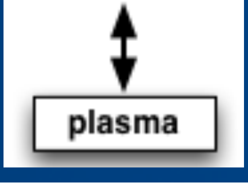


How can this current go negative?

(Analysis of frequency response data in shots 112230-112452 showed 1.5ms pure delay.)

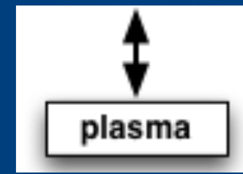
PLASMA MODEL VALIDATION

Plasma Model Validation

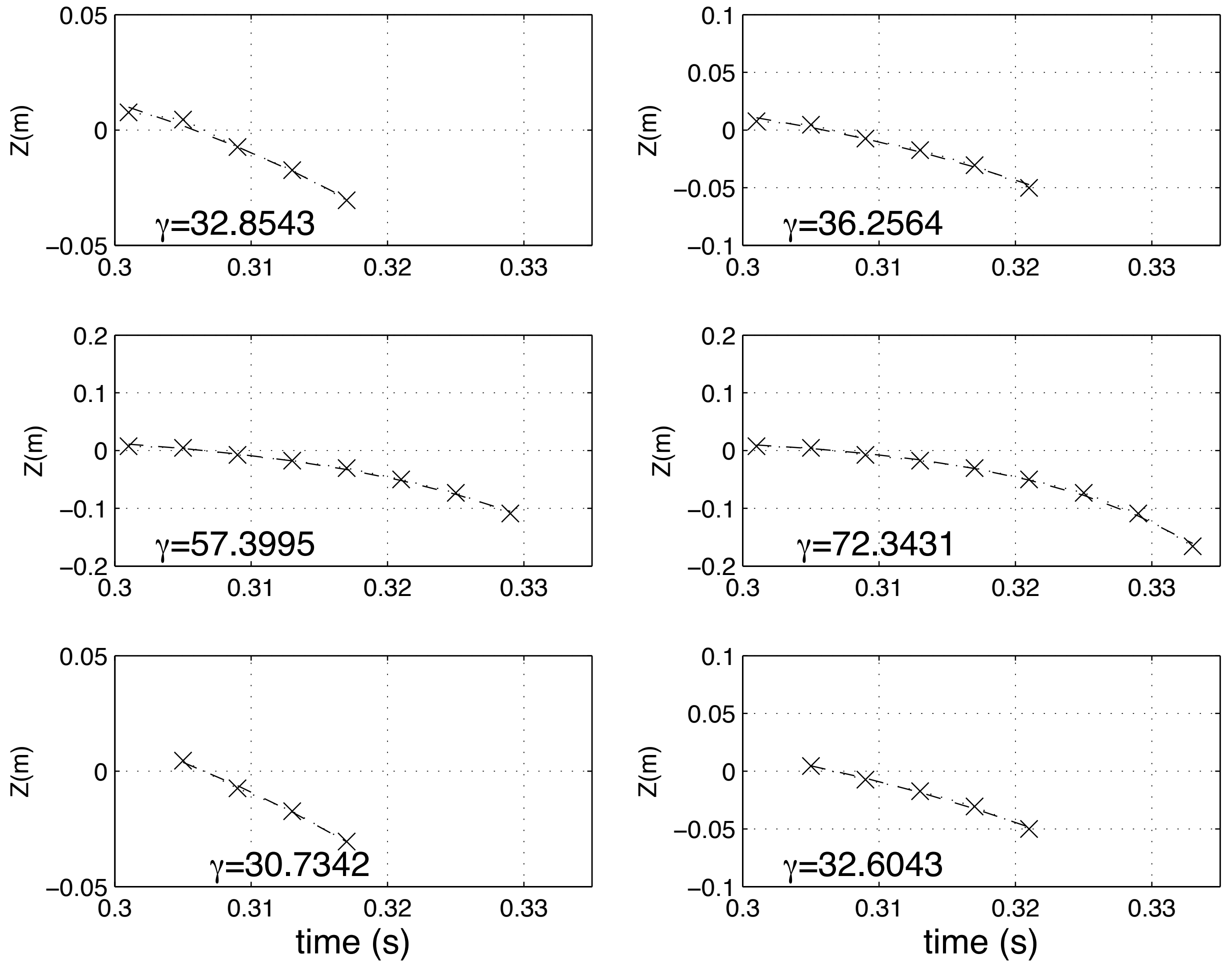


- **Verify that model correctly predicts growth rate of vertical instability.**
  - Compare model-predicted open-loop growth rate of vertical instability with exponential function fit to experimental data.
- **Verify that model correctly predicts variation of controlled parameters in response to chosen actuators (either voltage or current).**
  - Compare model-predicted boundary evolution with experimental data.
  - Quasi-static "perturbed equilibrium" model => slow variations in plasma shape sufficient for validation

Computing "measured" growth rates of vertical instability

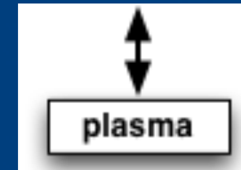


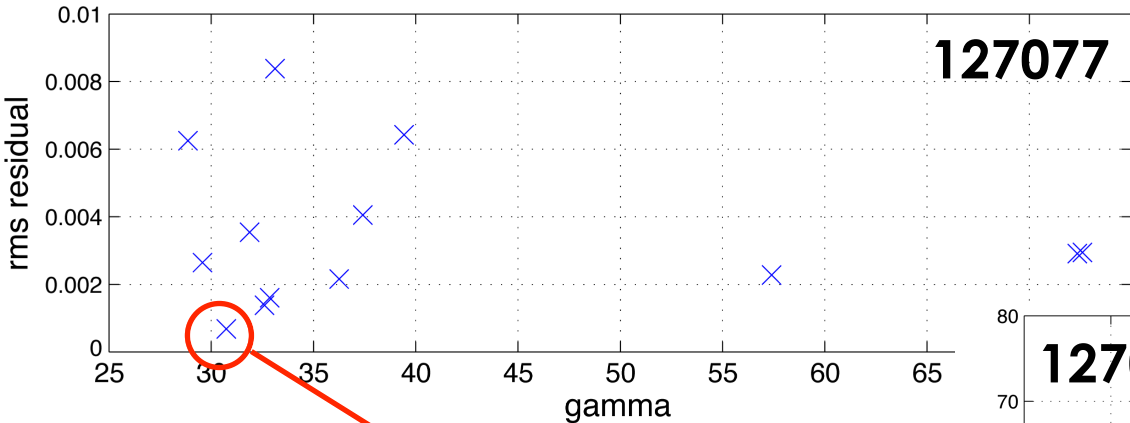
- **Fitting exponential function to 5ms sampled EFIT data.**
  - Fits shown all for shot 127077
  - Wide variation in fitted growth rate, depending on # samples and time window used.



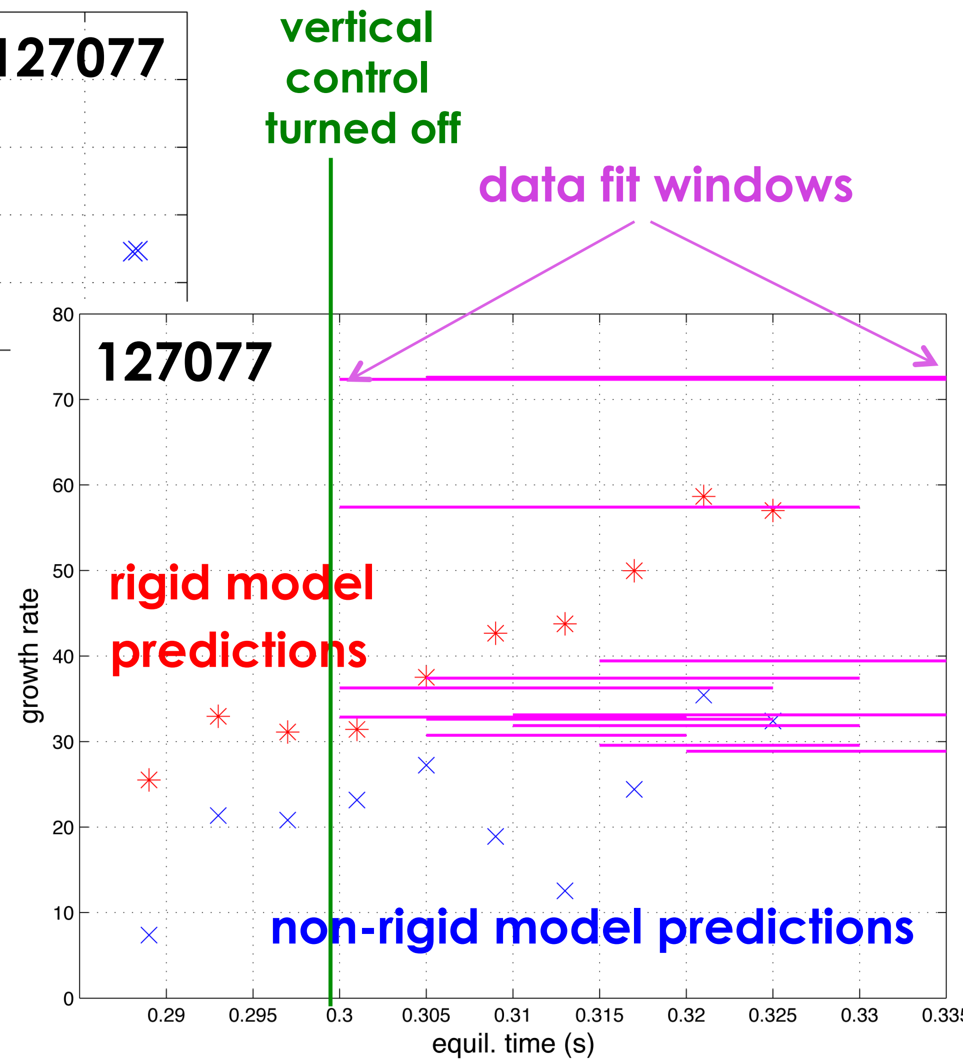
DIII-D
NATIONAL FUSION FACILITY

Comparing "measured" and model-predicted growth rates of vertical instability





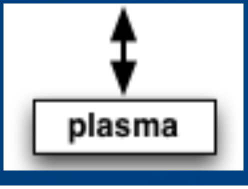
Select "true" growth rate by min RMS error of fit.



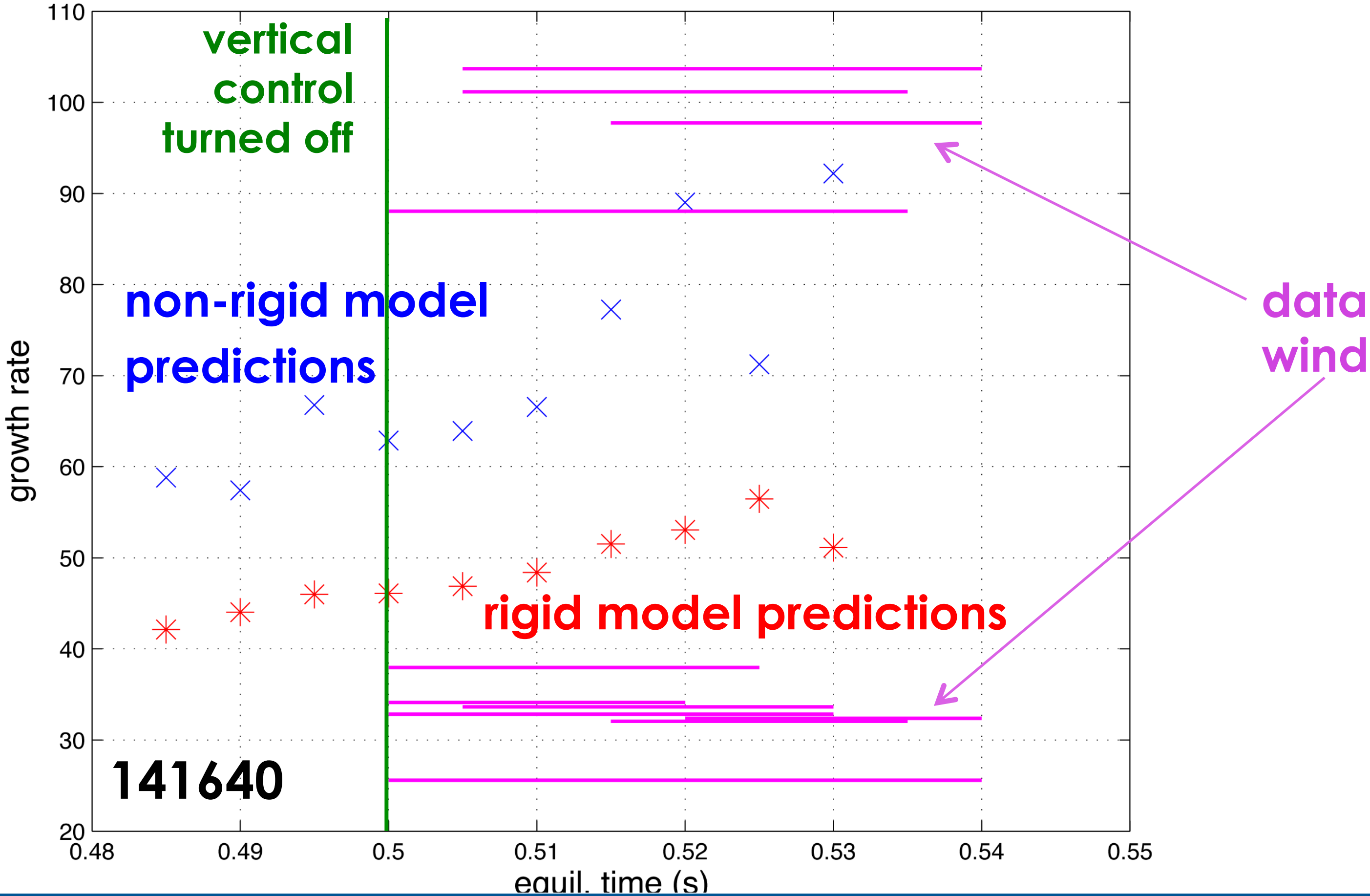
Fitted growth rate, rigid and non-rigid predictions reasonably consistent for 127074-87 series of shots.

DIII-D NATIONAL FUSION FACILITY

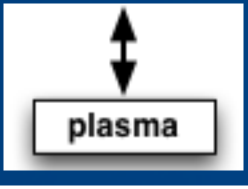
Comparing "measured" and model-predicted growth rates of vertical instability

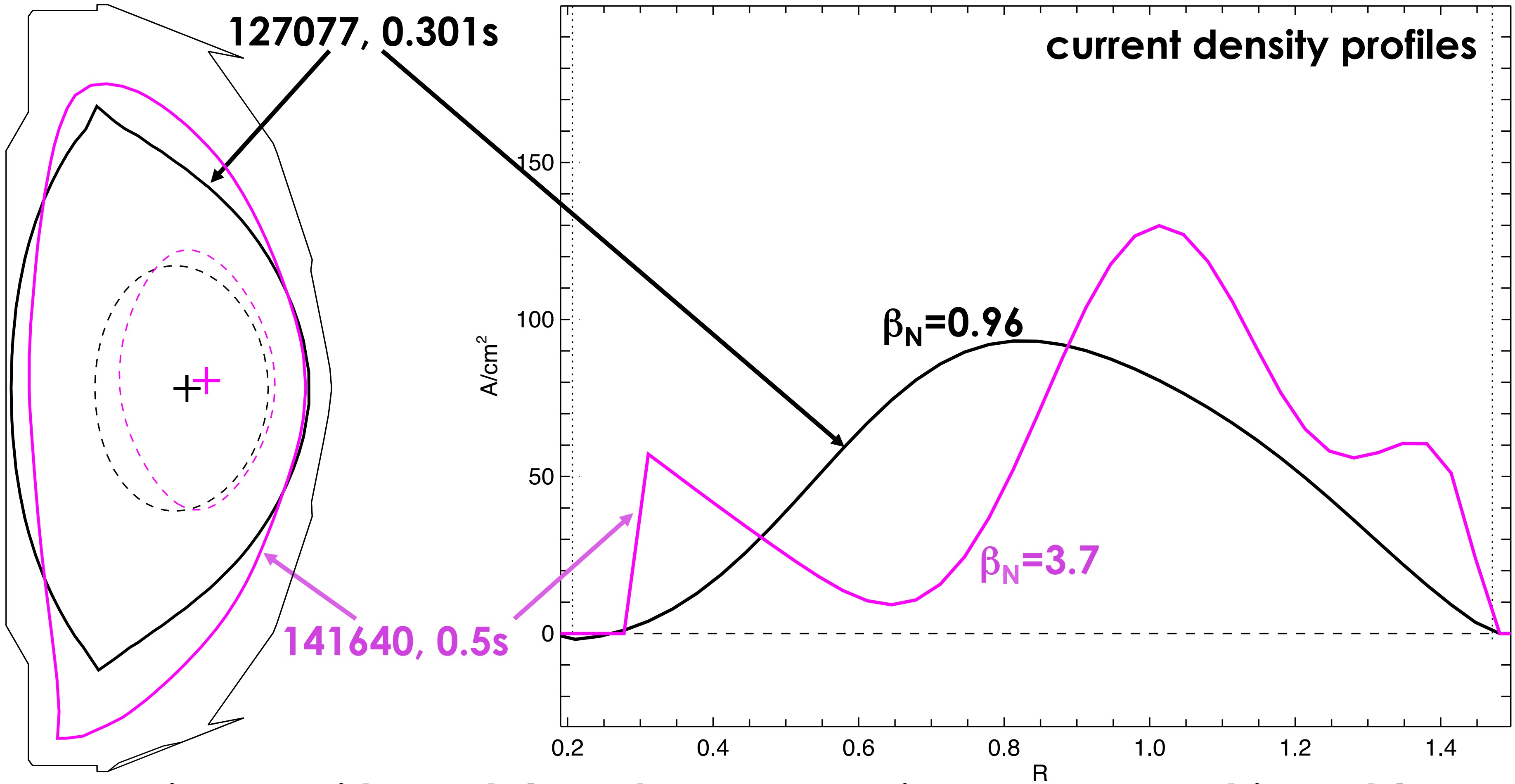


Fitted growth rate, rigid and non-rigid predictions NOT consistent for 141639-42 series of shots.



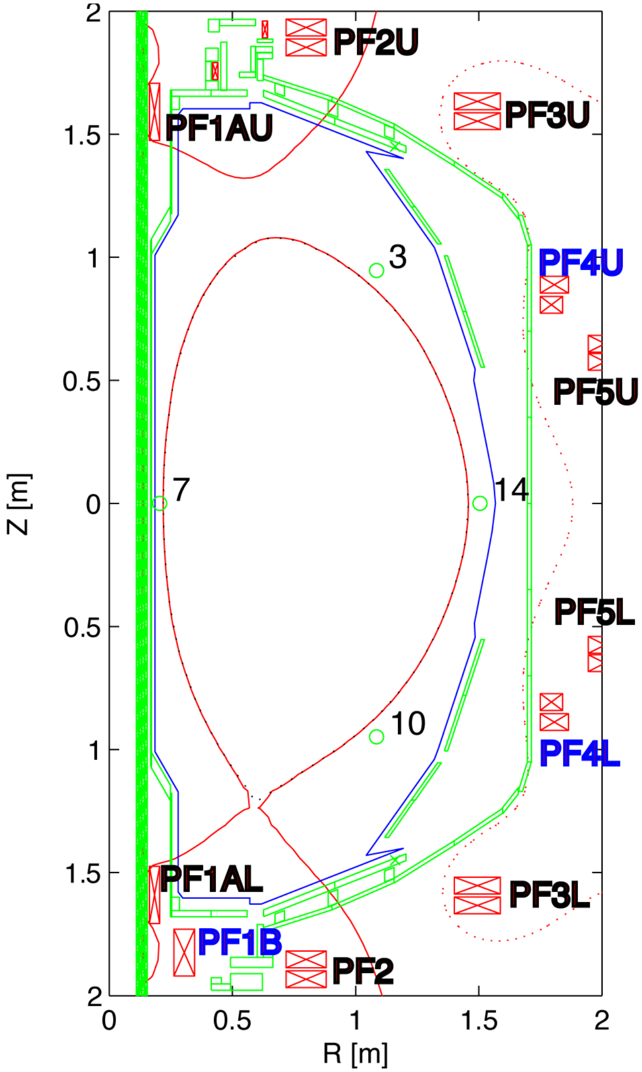
Model inconsistency seems correlated with high beta plasmas





- **Experience with model vs. data comparisons suggests this problem is a characteristic of low aspect ratio.**

Model-based control objective



- **Control NSTX boundary + Ip:**
  - 4 boundary points
  - 2 X-points (R and Z position of each)
  - Total plasma current Ip
  - Symmetry (relative flux between X-points)
  - **Total = 10 control parameters**
  - "Standard" control coils: OH, PF1AU, PF2U, PF3U, PF5, PF3L, PF2L, PF1AL (**Total 8**)

Steady-state map from coil currents to control parameters constructed using plasma response

boundary flux errors

reference flux (Ip control)

control params for X-points R

control params for X-points Z

symmetry

$$\begin{bmatrix} \psi_3 - \psi_{ref} \\ \psi_7 - \psi_{ref} \\ \psi_{10} - \psi_{ref} \\ \psi_{14} - \psi_{ref} \\ \hline \psi_{ref} \\ \hline B_{zX1} \\ B_{zX2} \\ \hline B_{rX1} \\ B_{rX2} \\ \hline \psi_{X1} - \psi_{X2} \end{bmatrix} = \begin{bmatrix} g_{1,1} & g_{1,2} & g_{1,3} & \cdots & \cdots & g_{1,11} \\ g_{2,1} & g_{2,2} & \ddots & & & \vdots \\ g_{3,1} & \ddots & \ddots & & & \\ \vdots & & & & & \\ \vdots & & & & & \\ g_{9,1} & & & & \ddots & \vdots \\ g_{10,1} & g_{10,2} & g_{10,3} & \cdots & \cdots & g_{10,11} \end{bmatrix} \begin{bmatrix} OH \\ PF1AU \\ PF2U \\ PF3U \\ PF4U \\ PF5 \\ PF4L \\ PF3L \\ PF2L \\ PF1AL \\ PF1B \end{bmatrix}$$

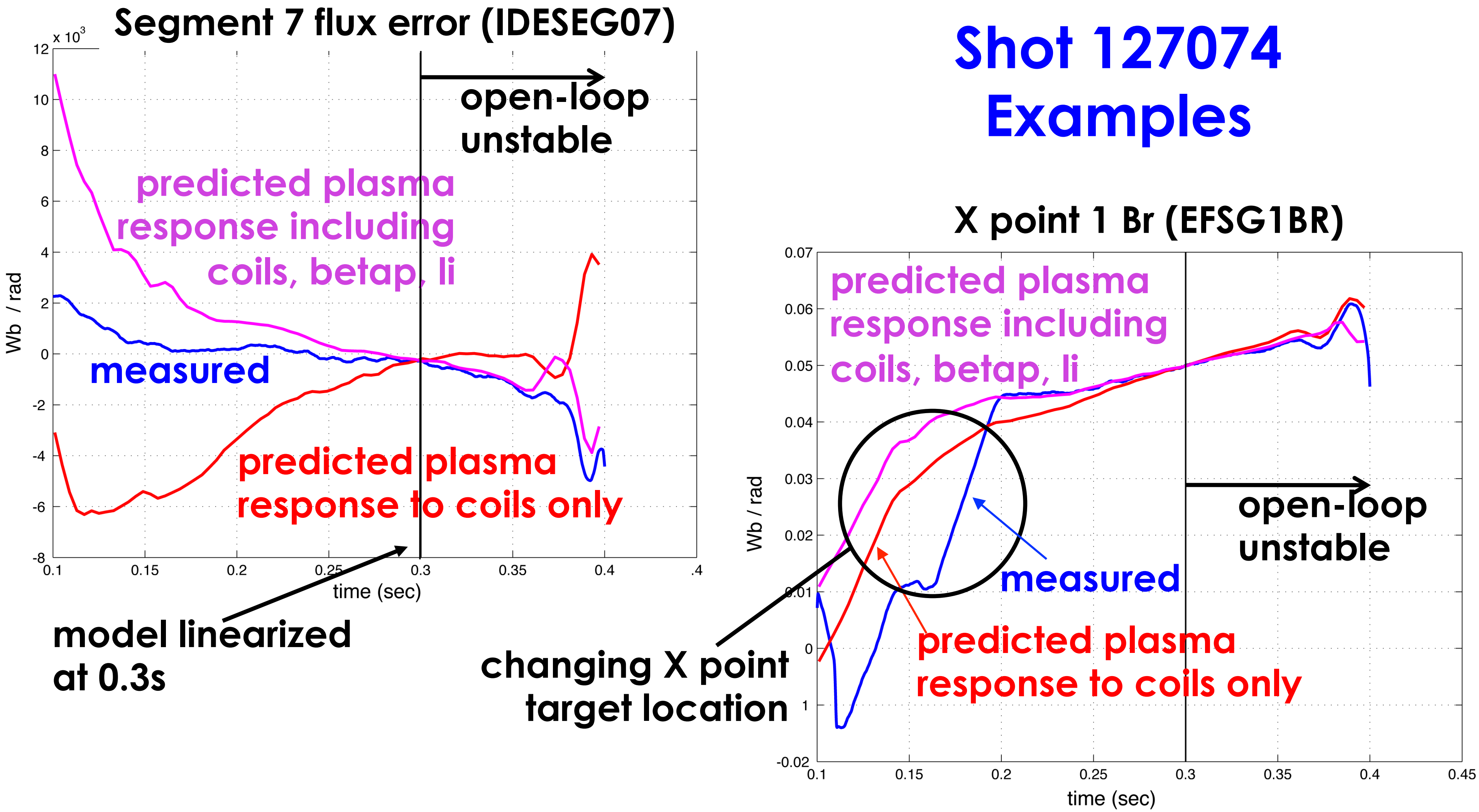
$$\mathbf{b} = \mathbf{G} \quad \mathbf{I_c}$$

PLASMA MODEL VALIDATION

Model-predicted versus experimental evolution of plasma boundary control parameters

plasma

Shot 127074 Examples



Model Validation Results Summary

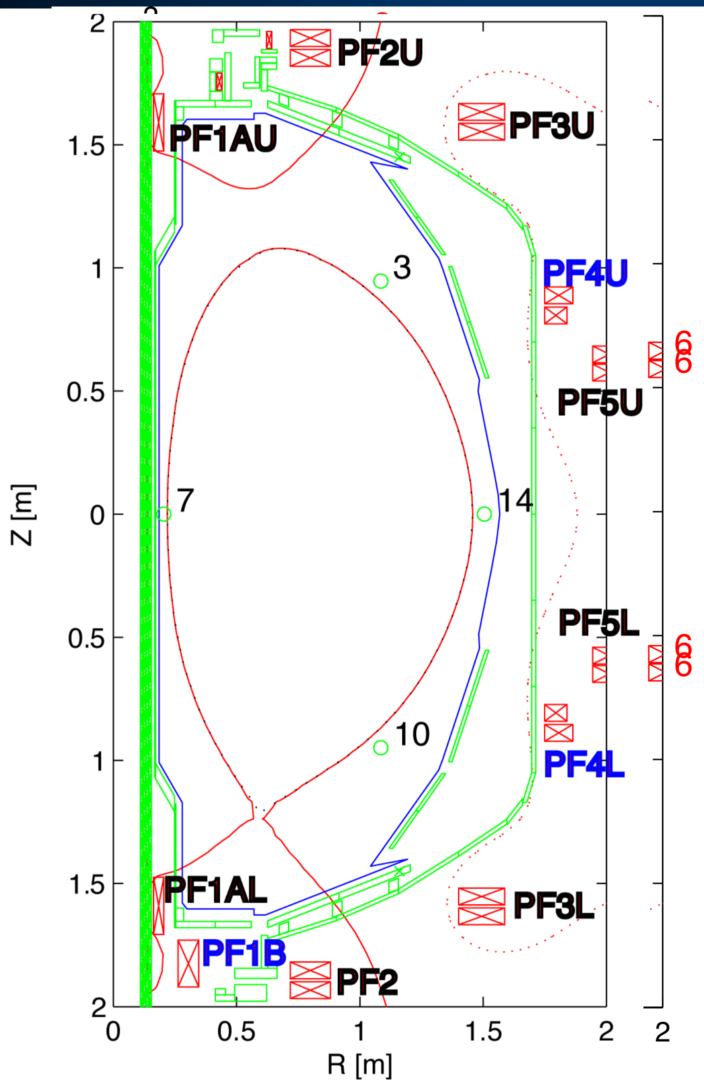
- Initial model responses V->I and I->y satisfactory, although would benefit from improvement.
- Power supply models mostly working well, with a couple of issues remaining.
- Vertical growth rates well-predicted for one (low beta) series of shots, but not for a second (high beta) series.
- Tokamak + plasma model appears predictive, but comparisons dominated by noise and disturbances (betap, li variation).

Further validation work needed

- **Collect vacuum sinusoid data near corner frequencies of coil and diagnostic model responses.**
- **Identify sources of gain / phase errors in vacuum coil and diagnostic responses.**
- **Develop fast-sampled Z estimate for fitting open-loop growth rates.**
- **Identify source of difference in rigid and non-rigid calculated growth rates for 141* series of shots.**
- **Collect experimental data for large controlled shape changes.**
  - Need large enough to dominate noise and (betap, li) disturbance effects.

CONTROLLABILITY ANALYSIS

Number of control parameters must be less than number of control coils



- **Can choose to reduce or combine control points**
  - Control 8 or fewer parameters using standard 8 control coils
- **Or, can choose to use more control coils**
  - Control all 10 parameters using (e.g.) OH, PF1AU, PF2U, PF3U, PF4U, PF5, PF4L, PF3L, PF2L, PF1AL, PF1B (**11 coils**)

DIII-D NATIONAL FUSION FACILITY

Exploring model-based control using decoupling controllers

- **Decoupling controller is "inverse" of mapping from coil currents to (isoflux + Ip) control parameters.**
  - Mapping from coil currents to isoflux errors + Ip:

$$GI_c = b$$

- **Simple to calculate and understand.**
  - Decoupling control gain matrix = pseudo-inverse of G

$$I_c \approx G^{+} b$$

- **Identifies controllability in steady-state.**
  - Neglects coil and passive conductor dynamics

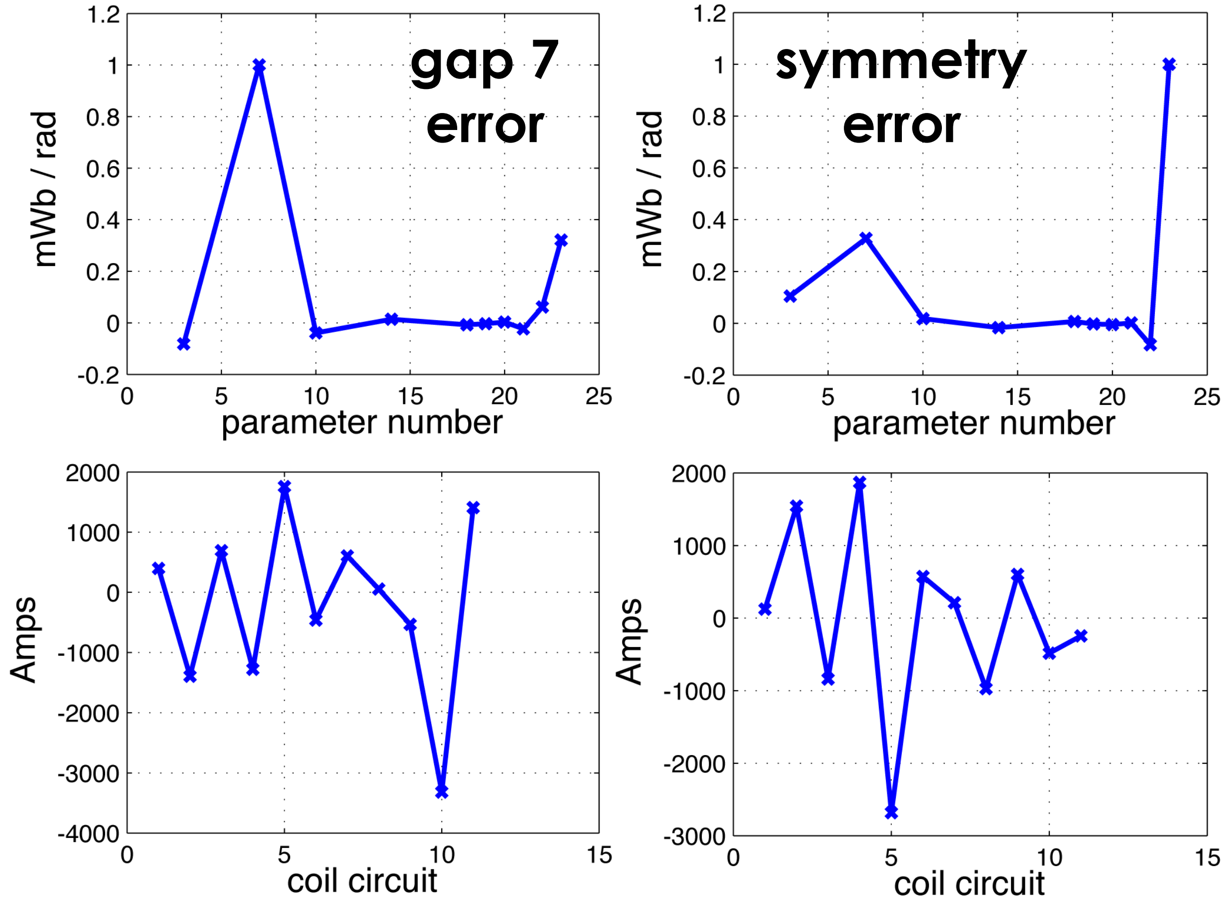
Condition number of mapping G reflects system (steady-state) controllability

$$\underbrace{\begin{bmatrix} \psi_3 - \psi_{ref} \\ \psi_7 - \psi_{ref} \\ \psi_{10} - \psi_{ref} \\ \psi_{14} - \psi_{ref} \\ \psi \;\; \cdots_{re} \\ B_{zX1} \\ B_{zX2} \\ B_{rX1} \\ B_{rX2} \\ \psi_{X1} - \psi_{X2} \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} g_{1,1} & g_{1,2} & g_{1,3} & \cdots & & \cdots & g_{1,11} \\ g_{2,1} & g_{2,2} & \ddots & & & & \vdots \\ g_{3,1} & \ddots & \ddots & & & & \\ \vdots & & & & & & \\ & & & & & & \\ \vdots & & & & & & \\ g_{9,1} & & & & & \ddots & \vdots \\ g_{10,1} & g_{10,2} & g_{10,3} & \cdots & & \cdots & g_{10,11} \end{bmatrix}}_{G} \underbrace{\begin{bmatrix} OH \\ PF1AU \\ PF2U \\ PF3U \\ PF4U \\ PF5 \\ PF4L \\ PF3L \\ PF2L \\ PF1AL \\ PF1B \end{bmatrix}}_{I_c}$$

- **In theory, matrix G is full rank => 10 control parameters can be controlled.**
- **In practice, condition number of G (~1000) => control will be difficult.**

Controlling inner gap is difficult even with all control coils

- **Control response still not completely decoupled**
- **Even this much decoupling requires unrealistically large current changes**



gap 7 disturbances large compared with coil response:

$$\partial\psi/\partial\beta_p = -1.7\, mWb/rad, \quad \partial\psi/\partial l_i = -1\, mWb/rad$$

Conclusions

- **Validated models:**
  - Enable model-based controller design => improved control => improved physics performance
  - Enable analysis of system controllability
  - Support design studies for NSTX-Upgrade
- **Initial validation is reasonably good**
  - Some improvement needed in component models
  - Validation process needs to made routine
    - "Life of device" activity: significant change in device or plasma scenario => change in models => revalidation
- **Supports proposed ITPA joint experiment MDC-18 (Evaluation of axisymmetric control aspects for ITER)**